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LETTER TO THE EDITOR

Phase transitions in nonlinear Abelian Higgs models

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Abstract. Phase transitions in O(n) nonlinear σ models coupled to an Abelian gauge field are studied near two dimensions. For all $n>0$, the transition is governed by the renormalisation-group fixed point of the CP^{2n-1} model. There is no fluctuation-induced first-order transition.

There have been a number of studies of phase transitions in Abelian Higgs models, defined by a Euclidean Lagrangian density

$$\mathscr{L}=\tfrac{1}{2}(\partial_\mu+\mathrm{i}eA_\mu)\phi^*_\alpha(\partial_\mu-\mathrm{i}eA_\mu)\phi_\alpha+V(\phi^*_\alpha\phi_\alpha)+\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{1}$$

In these models, a scalar field with $\frac{1}{2}n$ complex components $\phi_\alpha=\phi^1_\alpha+\mathrm{i}\phi^2_\alpha$ has a self-interaction described by the potential $V(\phi^*_\alpha\phi_\alpha)$ and is coupled to a single gauge field A_μ, whose field strength tensor has the usual form $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$. Summations over $\mu,\nu=1\ldots d$ and $\alpha=1\ldots\frac{1}{2}n$ are implied in (1). For spatial dimensionality d near four, and with a self-interaction given by

$$V(X)=\tfrac{1}{2}rX+(1/4!)uX^2, \tag{2}$$

perturbation theory suggests that the phase transition which occurs as r passes through a critical value $r_c(u,e)$ is driven weakly first order by fluctuations whenever $n<n_0(d)\approx 365.9+\mathrm{O}(4-d)$, while for $n>n_0$ it may be second order for $u\gtrsim\text{constant}\times e^2$ (Halperin *et al* 1974, Chen *et al* 1978, Lawrie 1982a, see also Coleman and Weinberg 1973). The first-order character is inferred essentially from the absence or inaccessibility of an infrared-stable fixed point of the renormalisation group. However, the construction of a free energy which explicitly verifies such behaviour can be carried through in practice only when the electric charge e is sufficiently small (Lawrie 1982b). Moreover a Monte Carlo simulation of the two-component model ($n=2$) in three dimensions provides no evidence for a first-order transition (Dasgupta and Halperin 1981). This may indicate that, at least for the value of the charge used in the simulation, some mechanism inaccessible to perturbation theory is responsible for the continuous nature of the transition, or else that $n_0(3)$ is actually less than two. It could also be that the apparently first-order character near four dimensions is merely an artefact of low-order perturbation theory.

In pursuit of this question, we have studied the nonlinear realisation of these models in $d=2+\varepsilon$ dimensions. That is, we replace V by the constraint $\phi^*_\alpha\phi_\alpha=1$, and use the partition function

$$Z=\int\mathscr{D}\phi\,\mathscr{D}A\prod_x\delta[\phi^*_\alpha(x)\phi_\alpha(x)-1]\exp\left(-t^{-1}\int\mathrm{d}^dx\,\mathscr{L}\right) \tag{3}$$

where t is now the temperature-like variable, and $\mathscr{L}$ is given by (1) with V omitted. Formally, this is equivalent to taking the limit $u=-6rt\to\infty$ in the potential (2). On general grounds of universality, we expect this model to exhibit the same phase transitions as that defined by (2) except possibly in regions of the (u, e^2) plane which are inaccessible to renormalisation-group flows from the region of large u. However, the perturbation method available for dealing with the nonlinear model takes the form of a low-temperature expansion combined with an expansion in powers of $\varepsilon=d-2$ (Brézin and Zinn-Justin 1976). It is a quite different type of approximation from the expansion in powers of u, e^2 and $(4-d)$ which is available for the linear model. Consequently, evidence for a first-order transition in the nonlinear models near two dimensions would greatly strengthen our confidence that the effect is genuine. *We find no evidence of a first-order transition.*

When $e=0$, our model reduces to the $\mathrm{O}(n)$-symmetric nonlinear σ model. Renormalisation-group analysis (Brézin and Zinn-Justin 1976) reveals a fixed point which is infrared-unstable to temperature perturbations from the critical value $t^*=\varepsilon/(n-2)+\mathrm{O}(\varepsilon^2)$. In the limit $e\to\infty$, the substitution $A_\mu=e^{-1}(B_\mu-\frac{1}{2}\mathrm{i}\phi_\alpha^*\overleftrightarrow{\partial}_\mu\phi_\alpha)$ yields the $CP^{\frac{1}{2}n-1}$-symmetric nonlinear σ model with B_μ decoupled,

$$\mathscr{L}\to\tfrac{1}{2}\partial_\mu\phi_\alpha^*\partial_\mu\phi_\alpha+\tfrac{1}{8}(\phi_\alpha^*\overleftrightarrow{\partial}_\mu\phi_\alpha)^2+\tfrac{1}{2}B_\mu B_\mu \qquad (4)$$

which has an infrared-unstable fixed point $t^{**}=\varepsilon/n+\mathrm{O}(\varepsilon^2)$ (Hikami 1979). Using the method described below, we have extended this analysis, at first order in ε, to the entire (e^2, t) plane. If fluctuation-induced first-order transitions were to occur, we would expect to find some range of values of e^2 for which the transition temperature lay outside the domains of attraction of both these fixed points. We find that the neutral model is indeed unstable to the gauge coupling at all temperatures. However, for all $n>0$, there is a smooth separatrix which divides the (e^2, t) plane into a high-temperature and a low-temperature region (figure 1). It terminates at the CP^{2n-1} fixed point which attracts renormalisation-group flows along it. When the neutral fixed point exists, i.e. for $n>2$, it forms the beginning of the separatrix.

Even in the two limiting cases, it is no easy matter to demonstrate explicitly that symmetry is restored above the fixed point (see, however, discussions by Jevicki (1978),

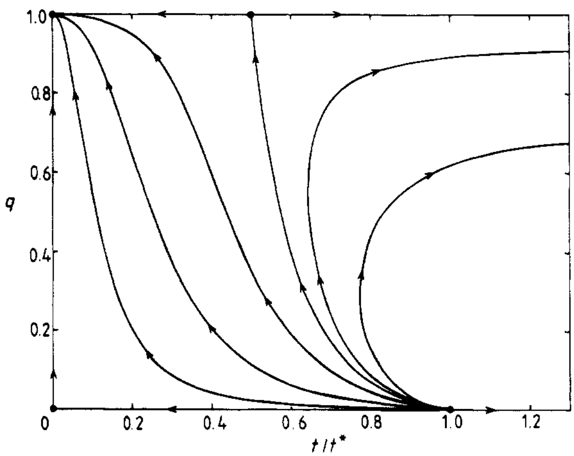

Figure 1. Renormalisation-group flows for $n=4$ and $\varepsilon=1$. Arrows indicate flow towards the infrared.

McKane and Stone (1980) and Amit and Kotliar (1980) for partial clarification). In the low-temperature phase, however, the existence of the fixed point suffices to obtain, at least formally, an order parameter and an inverse correlation length which vanish continuously as the fixed point is approached (Brézin and Zinn-Justin 1976). Given that the fixed points are indeed critical points, it follows that the separatrix in figure 1 is a line of critical points governed, except for $e^2=0$, by the CP^{2n-1} fixed point.

The main steps in our calculation are as follows. We first write the scalar fields as $\{\sigma \exp(\mathrm{i}\theta), \pi^\alpha\}$ where σ and θ are real and π^α ($\alpha = 2 \ldots \frac{1}{2}n$) are the complex components transverse to the direction of ordering. The partition function may then be written as

$$Z = \int \mathscr{D}(\sigma^2)\, \mathscr{D}\theta\, \mathscr{D}\pi\, \mathscr{D}A \prod_x \delta[\sigma^2(x) + \pi^2(x) - 1] \exp\left(-t^{-1} \int \mathrm{d}^d x\, \mathscr{L}_{\text{eff}}\right). \tag{5}$$

Taking into account the constraint $\sigma^2 + \pi^2 = 1$, we write the effective Lagrangian as

$$\begin{aligned}\mathscr{L}_{\text{eff}} = {} & \tfrac{1}{2}(\partial_\mu + \mathrm{i}eA_\mu)\pi^*_\alpha(\partial_\mu - \mathrm{i}eA_\mu)\pi_\alpha + \tfrac{1}{2}|(\partial_\mu - \mathrm{i}eA_\mu)(1-\pi^*_\alpha\pi_\alpha)^{1/2}|^2 \\ & + \tfrac{1}{2}(1-\pi^*_\alpha\pi_\alpha)(\partial_\mu\theta)^2 - e(1-\pi^*_\alpha\pi_\alpha)A_\mu\partial_\mu\theta \\ & + \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + (2\xi)^{-1}(\partial_\mu A_\mu - \xi\alpha e\theta)^2 H(1-\pi^*_\alpha\pi_\alpha)^{1/2}\cos\theta\end{aligned} \tag{6}$$

which now includes a gauge-fixing term proportional to $(2\xi)^{-1}$ and a source H coupled to ϕ^1_1. Inside the gauge-fixing term, the parameter $\alpha = 1 + \mathrm{O}(t)$ is chosen so as to eliminate, at each order of perturbation theory, the zero-momentum component of the Green function which connects A_μ and θ. Note that the gauge-fixing term we have chosen may be introduced without compensating ghost fields and also that with our choice of fields, σ and θ, no Jacobian arises from the elimination of σ. In the limit $e^2 \to \infty$ with $\xi \neq 0$, the gauge condition becomes $\theta = 0$.

The one-loop graphs which contribute to the various two-point Green functions are shown in figure 2. The function connecting A_μ and θ, figure 2(a), depends on the external momentum only through an overall factor p_μ at this order and can be made to vanish by a suitable choice of α in (5). For finite values of e^2, renormalisation is accomplished by minimal subtraction of poles at $\varepsilon = 0$. In the limit $e^2 \to \infty$, the independent fields which survive are π^α and B_μ. We ensure that their propagators remain finite, order by order in ε and t, by subtracting also appropriate powers of $\ln(1+e^2)$ and $\ln(1+\xi e^2)$. Details of this scheme will be described in a separate

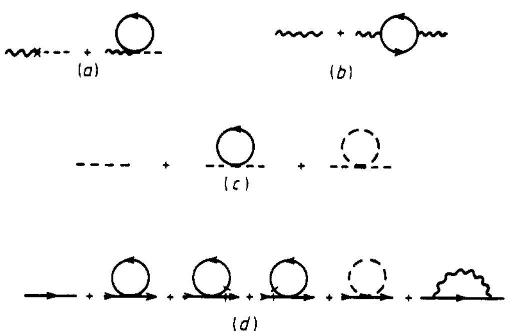

Figure 2. One-loop contributions to two-point Green functions. (a) the A_μ–θ propagator, (b) the photon propagator, (c) the θ propagator, (d) the π^*–π propagator. A stroke (—+—) indicates a factor of p_μ, the momentum carried by the line.

publication: an analogous procedure is described in Lawrie (1981, 1982a). In particular, the temperature and electric charge are renormalised according to

$$t_0 = \mu^{-\varepsilon} t[1 + (n-2)t/\varepsilon - t\ln(1+e^2) + O(t^2)] \tag{7}$$

$$e_0^2 = \mu^2 e^2[1 - (n-2)t/\varepsilon + O(t^2)] \tag{8}$$

where now, on the left, subscripts denote the bare parameters appearing in (5) and (6). The arbitrary renormalisation mass scale μ is introduced to make t and e dimensionless, and the usual geometrical factor $2\pi^{d/2}/(2\pi)^d(\frac{1}{2}d-1)!$ has been absorbed into t. At first order of the double expansion in powers of t and ε, the corresponding renormalisation-group functions are

$$W(t, e^2) \equiv \mu\partial t/\partial\mu|_{e_0 t_0} = \varepsilon t - [n - 2/(1+e^2)]t^2 \tag{9}$$

$$\gamma_e(t, e^2) \equiv \mu\partial \ln e^2/\partial\mu|_{e_0 t_0} = 2 - (n-2)t. \tag{10}$$

To accommodate the limit $e^2 \to \infty$, we define $q = e^2/(1+e^2)$. Then the evolution of the effective temperature $\bar{t}(\lambda)$ and charge squared $\bar{q}(\lambda)$ under a change of momentum scale by a factor λ is governed by the renormalisation-group flow equations

$$\lambda\partial\bar{t}/\partial\lambda = \varepsilon\bar{t} - (n-2+2\bar{q})\bar{t}^2 \tag{11}$$

$$\lambda\partial\bar{q}/\partial\lambda = -\bar{q}(1-\bar{q})[2-(n-2)\bar{t}] \tag{12}$$

with $\bar{t}(1) = t$ and $\bar{q}(1) = q$. Some trajectories generated by these equations are illustrated in figure 1 for the case $n = 4$ and $\varepsilon = 1$, where arrows indicate the direction of flow in the infrared limit $\lambda \to 0$. There are zero-temperature fixed points at $q = 0$ and $q = 1$ associated with Goldstone mode singularities in the neutral and charged models respectively. At non-zero temperatures there are the two known fixed points at $(q, t) = (0, \varepsilon/(n-2))$ and $(q, t) = (1, \varepsilon/n)$. The former recedes to $t = \infty$ at $n = 2$, reflecting the special properties of the neutral two-dimensional XY model. However, except for $n = 0$, the separatrix represents a finite critical temperature $t_c(q)$ for all non-zero q.

We conclude that, except possibly in the limit $n \to 0$, addition of a gauge field to the O(n) model does not lead to a first-order transition near two dimensions.

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